The Effect of the Radon Transform on the MR Image Compression

A comparative study

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Abstract—The medical image accumulated over time for the purpose of research confronts us with the problems of transmission and storage. So an efficient compression scheme is necessary to eliminate this serious problem. We present in this paper a Magnetic Resonance Imaging scans compression method based on the Radon transform and DCT with high quantification. This method is a modification of the scheme presented by S. A Pradeep and R. Manavalan. A comparative study is performed to show the power of the Radon transform against the quantization noise.

Index Terms—Image compression, Magnetic Resonance Imaging (MRI), Radon transform, Integration process, Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT).

I. INTRODUCTION

Medical imaging such as CT scanners and MRI machines allow the detection of cancers, Diagnosing lung and chest problems, and examining soft tissue ligament and tendon injuries in spinal cord injuries and brain tumors, etc. These images will be stored for monitoring the health status of patients for long periods. These medical images can also be transmitted to other specialized medical center for deeper examination. The storage and transmission of these pictures need important memory resources and high bandwidth, hence the necessity of effective image compression methods for solving these two major problems. There are two classes of images compression: lossless compression and lossy compression. Lossless compression allows reconstruction after decompression, an identical image to the original but with a low compression ratio. Among the most widely used coders, we distinguish Huffman coding [1], the arithmetic coding, Golomb-Rice coding [2], the Tunstall coding [3] and the Linear Predictive Coding (LPC) [4]. These encoders are distinguished from the lossy compression, which introduces an irreversible degradation of the original image but allows much greater compression than that obtained by lossless compression methods. These methods are generally based on the quantization of blocks of coefficients resulting from a transform. Among of these transforms: The Karhunen-Loeve Transform (KLT), the DCT Transform [5], the Radon transform [6]-[7], the fractal Transform [8], the Discrete Walsh-Hadamard Transform (DWHT), the Discrete Wavelet Transform (DWT), which forms the basis of various techniques like EZW algorithm [9], the SPIHT algorithm [10] and the JPEG2000 standard [11]. We also distinguish the lossy predictive coding such as DPCM (Differential Pulse Code Modulation) [12].

We show through this paper the power of the Radon transform by means of the of image compression method based on the Radon transform and a high quantization of DCT, which is a modification of the method developed by Pradeep and Manavalan [7]. A comparative study with several techniques is performed to show the effectiveness of this compression scheme.

The Radon transform has become a very important tool in the field of image compression seen its robustness against different noises such as white noise and the quantization noise compared to other types of transformations as the Fourier transform and wavelet transform [13]. It is particularly used for detecting objects in noisy images like in the case of ship wake detection in radar images [14].

Ahmed, Natarajan and Rao [5] have showed that the DCT has a close compression ability to that of the KLT. It is also nearly equal to the KLT in its ability of energy compaction. Unlike KLT, the DCT is independent of the image in question, that is to say, the core of its matrix is set at a given size.
Therefore, any information about the blocks size at the receiver side is required for the reconstruction of the original image.

The rest of this paper is organized as follows: Section 2 is devoted to the theory of the image compression method used in this paper. Section 3 gives the different results of the evaluation criteria used in the comparison and their discussions. Finally, Section 4 summarizes the entire paper with a conclusion.

II. THE PROPOSED METHOD

This method is a modification of the method developed by Pradeep and Manavalan [7]. This approach exploits the Radon transform and DCT with high scale of quantization, which is 20 for MR image compression. This method begins with the application of the Radon transform to the original image to obtain the Radon points. The next step consists to encode the Radon points using a high quantization of the DCT. These steps in reverse order allow to reconstruct the original image.

The Radon transform is used to represent an image in the Radon field as a collection of projections along different directions for each given angle [6]. In other words, the Radon transform maps a line to a point in the Radon domain; a particular line to a particular point. One of the most important properties of the Radon transform is that it is reversible hence the possibility of reconstructing the image from the knowledge of its integrations along hyperplanes of its space.

The Radon transform is applied to an input image \( I \) in the range \( \theta \ [0, 2\pi] \) to collect all of its projection along a specific direction in the \( x \) and \( y \) axis domain. All projections of the image \( I \) is given by equation 1.

\[
\hat{I}_\theta(p) = \int_{-\infty}^{+\infty} I(\rho) \delta(p - \rho \cos \theta - \rho \sin \theta) d\rho. \tag{1}
\]

With:
- \( I \) : Input image
- \( \rho \) : Vector of position of the components \((x_1,x_2,\ldots,x_n)\).
- \( p = \rho \hat{\rho} \) : A hyperplane, with \( \hat{\rho} \) is a unit vector.
- \( \delta \) : Dirac function.

The graphical representation of the Radon transform gives a sinogram. Fig. 2 gives an example of the representation of the Radon transform of MR head image.

The Discrete Cosine Transform (DCT) takes its name from the fact that the rows of the matrix of the transform of size \( N \times N \) are obtained by function of cosine as shown in the equation 2 [16].

\[
C(i,j) = \begin{cases} 
\frac{1}{\sqrt{N}} & i=0, j=0,\ldots,N-1 \\
\frac{2}{\sqrt{2N}} \cos \frac{(2j+1)\pi i}{2N} & i=0,\ldots,N, j=0,1,\ldots,N-1.
\end{cases} \tag{2}
\]

With:
- \( C(i,j) \) : The DCT transform coefficient at the position \( (i,j) \).

The quantization matrix that we use in our case is represented as follows:
This matrix is obtained by multiplying the standard matrix used in the JPEG standard by 20. This matrix provides a quality level of 50 and renders both high compression and excellent decompression image quality. The standard quantization matrix is represented as follows:

\[
Q(i,j) = \begin{bmatrix}
320 & 220 & 200 & 320 & 480 & 800 & 1020 & 1220 \\
240 & 240 & 280 & 380 & 520 & 1160 & 1200 & 1100 \\
280 & 260 & 320 & 480 & 800 & 1140 & 1380 & 1120 \\
280 & 340 & 240 & 580 & 1020 & 1740 & 1600 & 1240 \\
360 & 440 & 740 & 1120 & 1360 & 2180 & 2060 & 1540 \\
480 & 700 & 1100 & 1280 & 1620 & 2080 & 2260 & 1840 \\
980 & 1280 & 1560 & 1740 & 2060 & 2420 & 2400 & 2020 \\
1440 & 1840 & 1900 & 1960 & 2240 & 2000 & 2060 & 1980 \\
\end{bmatrix}
\]

A coefficient is quantized by the following equation:

\[
D(i,j) = \text{Round}\left( \frac{C(i,j)}{Q(i,j)} \right),
\]

With:

\(C(i,j)\): Is the DCT transform coefficients.

\(Q(i,j)\): The corresponding element in the quantization matrix.

The DCT coefficients are restored with the quantization error by the following equation:

\[
C(i,j) = D(i,j) \times Q(i,j)
\]

One of the drawbacks of image compression using the DCT is the pixilation effect. When a high quantization is applied, the block division becomes visible because an entire block is encoded with the same value (few non-zero coefficients to represent). We will use this compression technique in our comparison with a high level of quantization to quantify the difference between the two methods.

**III. RESULTS AND DISCUSSIONS**

We conducted a comparative study that includes two methods: image compression method using biorthogonal wavelet [17] and the proposed method. We have used one MR head image with resolution of 128 x128 and 8 bpp [15].

We have used the PSNR and the compression ratio to evaluate the methods and quantify the difference between them.

The compression ratio allows evaluating the efficacy and potency of the image compression method. The compression ratio is given by the following equation.

\[
\text{CR} = \frac{\text{Number of bits in the original image}}{\text{Number of bits in the compressed image}}
\]

PSNR measures the distortion introduced by the compression operation. The PSNR is given by the following equation.

\[
\text{PSNR}(I_0, I_c) = 20 \log_{10} \left( \frac{2^r - 1}{\sqrt{\text{MSE}(I_0, I_c)}} \right)
\]

Where \(I_0\) and \(I_c\) represents respectively the original image and the reconstructed image of size \(M \times N\). and \(r\) represents the numerical resolution of the image. \(r = 8\) in our case.

MSE is the Mean Square Error.

The different results of the evaluation criteria are listed in the Table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR (dB)</th>
<th>CR</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biorthogonal wavelet</td>
<td>32.57</td>
<td>3.0</td>
<td>[17]</td>
</tr>
<tr>
<td>Bior 3.3 filter</td>
<td>35.50</td>
<td>7.27</td>
<td></td>
</tr>
<tr>
<td>The proposed method</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

We notice from the table that the proposed method provides an excellent compromise rate-distortion despite the high quantification. It allows to obtain a compression ratio of 7.27, which is higher by 4.27 to that obtained by the DWT method. We also note that the quality of the image reconstructed by the proposed method is of better quality with a PSNR of 35.50, which is higher by 2.93 to that obtained by the DWT method.

These results are provided by the robustness of the Radon transform against the quantization noise.

Murphy showed that the Radon transform attenuates the intensity fluctuations by the integration process. This process comes from the fact that the inverse Radon transform is obtained using Filter-BackProjection algorithm (FBP). The FBP allows to filter each Radon point by the Ram-Lak filter [18].

**IV. CONCLUSION**

We have shown through this article the effect of the Radon transform on the compression of MR images. For this, we have...
made a change to the scheme developed by S. A. Pradeep and R. Manavalan. This modification consists to change the scale of quantification to 20 to ensure a strong compression. The results showed the power of the Radon transform against the quantization noise in order to realize a high compression of MR images while preserving the quality of these images after decompression.

REFERENCES


